Next-to-Minimal Supersymmetric Standard Model may be Minimal in F-theory

Kang-Sin Choi*
Korea Institute for Advanced Study, Seoul 130-722, Korea

The minimal supersymmetric Standard Model is sought in F-theory. After obtaining the gauge group $SU(3)\times SU(2)\times U(1)$ by specifying spectral cover, we choose a universal G-flux obeying $SO(10)$ unification relation. The requirement on *different* numbers between Higgs pairs and matter generations (respectively one and three) *forces* distinction between up and down Higgses, suppression of proton decay operators up to dimension five, and existence of a singlet related to μ-parameter.

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ROLE OF UNIFICATION IN STRING THEORY

Microscopic phenomena up to sub-TeV scales are successfully described by the Standard Model (SM). Its overlooked but important feature might be being a gauge theory based on a particular group. Quantum consistency of anomaly cancellation requires very special matter contents, and it is best understood by spontaneous symmetry breaking of a unifying theory extending the gauge group [1]. The minimal anomaly free chiral representation $\mathbf{16}$ of $SO(10)$ contains all the observed fermions of SM including right-handed neutrino. With the aid of supersymmetry (SUSY), Higgs bosons belong to a hypermultiplet and can be treated on equal footing as matter. Thus matter and Higgs pair (as well as colored Higgs pair) are unified to a single representation $\mathbf{27}$ of E_6, predicting another kind of singlet related to μ-parameter of SUSY [2]. Continuing, larger groups in higher dimension can even unify vector multiplets. This is well-known chain of E_n unifications in which the position of the SM group is unique $E_3\times U(1)$.

However, it is difficult to realize the unification in field theory, yielding the observed vacuum configuration by the scalar potential using only scalar fields. Magnetic flux or gauge bundle in extra dimensions supplements the idea, but anomaly cancellation is only guaranteed if it is embedded in consistent string theory, free of one-loop divergence. So we construct the model from heterotic string or its dual F-theory, which explains the origin of E_8 gauge theory where the E_n series ends [3], and provides all the above ingredients. Further constraints, such as repetition of matter generations and gauge coupling unification, show this unification pattern itself is compelling in any UV completion [4].

Our strategy here is to construct the SM gauge group from the beginning, nevertheless the matter generations and Higgses can follow unification relation of a larger group. This feature is clearly realized by spectral cover construction [5]. It enables us to explicitly construct (poly)stable vector bundle on elliptic fibered manifold, using two data. The first is spectral cover that specifies the structure group 'broken part of the E_8' hence determines unbroken gauge group, here to be $SU(3)\times SU(2)\times U(1)$. To obtain chiral spectrum in four dimension, we also have to turn on so called G-flux. If we *partly* turn on the flux on $S[U(3)_\perp\times U(1)]$ spectral cover, the resulting *identities* of chiral matter fall into $\mathbf{27}$ of E_6, the commutant of $SU(3)$ in E_8, and the *numbers* of matter generations and Higgs pairs obey a unification relation of $G=SO(10)$, the commutant to $SU(4)$. Same things happen in heterotic dual theory. Those two data are translated into a vector bundle $\mathcal{V}$ of H in the heterotic side by Fourier–Mukai transformation [6, 7]. So, first the vector bundle $\mathcal{V}$ of the same structure group yields the unbroken group G. Then there is a Wilson line on the elliptic fiber, breaking G down to the SM group. However, the language of spectral cover is easy to distinguish matter from Higgs and to control their multiplicities separately.

Finally, the four dimensional interactions follow from gauge invariant terms of the higher dimensional effective Lagrangian by dimensional reduction [8, 9]. The invariance under the mother group E_8 plays the role of selection rule.

GAUGE GROUP

The model is obtained from F-theory compactification on elliptic Calabi–Yau fourfold with a section, admitting heterotic dual. We construct the spectral cover for the structure group $S[U(5)\times U(1)_Y]$ as follows [10, 11]. First, embed a twofold B_2 into a projectivized threefold $\check{Z}=\mathbb{P}(K_{B_2}\oplus\mathcal{O})\xrightarrow{\pi} B_2$ using the canonical bundle K_{B_2} [6, 13] (Its global embedding using heterotic dual is easily done [7]). The spectral cover in it is described by

$$(b_0s^5+b_1s^4+b_2s^3+b_3s^2+b_4s+b_5)(s+a_1)=0 \quad (1)$$

where $a_1=-b_1/b_0$ and s is an affine coordinate parameterizing the normal direction to $B_2=\{s=0\}$. Each coefficient b_m, parameterizing the positions of the covers, is related to the elementary symmetric polynomial of degree m, out of weights of the fundamental representations $\mathbf{5}_1+\mathbf{1}_{-5}$ of the $S[U(5)\times U(1)_Y]$. The surviving group on B_2 is the commutant, the SM group $SU(3)\times SU(2)\times U(1)_Y$. This a sufficient specification, but it also provides the information on the unbroken gauge group [12].

The singularity equation corresponding to the SM

matter	matter curve	homology on B_2
$q(\mathbf{3},\mathbf{2};\mathbf{3})_{\frac16,1,1}$	$\prod t_i \to 0$	$\eta - 3c_1$
$u^c(\overline{\mathbf{3}},\mathbf{1};\mathbf{3})_{-\frac23,1,1}$	$\prod(t_i+t_6)\to 0$	$\eta - 3c_1$
$e^c(\mathbf{1},\mathbf{1};\mathbf{3})_{1,1,1}$	$\prod(t_i-t_6)\to 0$	$\eta - 3c_1$
$d^c(\overline{\mathbf{3}},\mathbf{1};\mathbf{3})_{\frac13,-3,1}$	$\prod(t_i+t_5)\to 0$	$\eta - 3c_1$
$l(\mathbf{2},\mathbf{1};\mathbf{3})_{-\frac12,-3,1}$	$\prod(t_i+t_5+t_6)\to 0$	$\eta - 3c_1$
$\nu^c(\mathbf{1},\mathbf{1};\mathbf{3})_{0,5,1}$	$\prod(t_i-t_5)\to 0$	$\eta - 3c_1$
$h_u^c(\mathbf{2},\mathbf{1};\overline{\mathbf{3}})_{\frac12,2,2}$	$\prod(t_i+t_j+t_6)\to 0$	$\eta - 3c_1$
$h_d(\mathbf{2},\mathbf{1};\mathbf{3})_{\frac12,2,-2}$	$\prod(t_i+t_4+t_6)\to 0$	$\eta - 3c_1$
$D_1^c(\overline{\mathbf{3}},\mathbf{1};\overline{\mathbf{3}})_{\frac13,2,2}$	$\prod(t_i+t_j)\to 0$	$\eta - 3c_1$
$D_2^c(\overline{\mathbf{3}},\mathbf{1};\mathbf{3})_{\frac13,2,-2}$	$\prod(t_i+t_4)\to 0$	$\eta - 3c_1$
$S(\mathbf{1},\mathbf{1};\mathbf{3})_{0,0,4}$	$\prod(t_i-t_4)\to 0$	$\eta - 3c_1$
$X(\mathbf{3},\mathbf{2};\mathbf{1})_{-\frac56,0,0}$	$t_6\to 0$	$-c_1$
$Y(\mathbf{3},\mathbf{2};\mathbf{1})_{\frac16,-4,0}$	$t_5\to 0$	$-c_1$
$T^c(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{-\frac23,-4,0}$	$t_5+t_6\to 0$	$-c_1$
$\Sigma(\mathbf{1},\mathbf{1};\mathbf{1})_{1,-4,0}$	$t_5-t_6\to 0$	$-c_1$
$Q(\mathbf{3},\mathbf{2};\mathbf{1})_{\frac16,1,-3}$	$t_4\to 0$	$-c_1$
$U^c(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{-\frac23,1,-3}$	$t_4+t_6\to 0$	$-c_1$
$E^c(\mathbf{1},\mathbf{1};\mathbf{1})_{1,1,-3}$	$t_4-t_6\to 0$	$-c_1$
$D^c(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{\frac13,-3,-3}$	$t_4+t_5\to 0$	$-c_1$
$L(\mathbf{2},\mathbf{1};\mathbf{1})_{-\frac12,-3,-3}$	$t_4+t_5+t_6$	$-c_1$
$N^c(\mathbf{1},\mathbf{1};\mathbf{1})_{0,5,-3}$	$t_4-t_5\to 0$	$-c_1$

TABLE I. Matter contents identified by $SU(3)\times SU(2)\times SU(3)_\perp\times U(1)_Y\times U(1)_X\times U(1)_Z$ quantum numbers. All the indices take different value in $S_3=\{1,2,3\}$. The fields below middle line are decoupled.

group is given by

$$\begin{aligned} y^2 = {} & x^3+(b_5+b_4a_1)xy+(b_3+b_2a_1)(a_1b_5+z)yz \\ & +(b_4+b_3a_1)x^2z+(b_2-b_0a_1^2)(a_1b_5+z)xz^2 \\ & +b_0(a_1b_5+z)^2z^3, \end{aligned} \tag{2}$$

where x,y are affine coordinates of $\mathbb{P}^2$ and z is the normal coordinate to $B_2=\{z=0\}$ inside the base of elliptic fibration B in F-theory side. At the discriminant locus Δ, we have the the SM gauge group [11]. Referring to Tate's table [12], already (2) is a special form of the $SU(3)$ singularity whose parameters are tuned up to $\mathcal{O}(z^5)$. A change of coordinate $a_1b_5+z\to z$ shows the other $SU(2)$ part also special up to $\mathcal{O}(z^5)$. The $U(1)_Y$ part is the relative position between two linearly equivalent components. The Calabi–Yau conditions require that the b_m are sections of $\eta-mc_1$, where $\eta=6c_1(B_2)+c_1(N_{B_2/B})$ and $c_1=c_1(B_2)$ are combinations of tangent and normal bundle to B_2. The leading order locus of Δ in z coincides with B_2.

To distinguish Higgs from lepton doublets, we need an extra $U(1)_X$ symmetry inside $U(5)$ cover, which is the famous continuous version of matter parity, a cousin of $U(1)_{B-L}$. Shortly we will see, for the *observed* number of Higgses in four dimension, we need one more parameter from an extra $U(1)_Z$, so that the structure group should be factorized as

$$S[U(3)_\perp\times U(1)_Z\times U(1)_X\times U(1)_Y]. \tag{3}$$

The resulting spectral cover, respectively $C_3\cup C_Z\cup C_X\cup C_Y$, is described by $(f_0s^3+f_1s^2+f_3s+f_4)(s+e_1)(s+d_1)(s+a_1)$ with the constraint $f_0(a_1+d_1+e_1)+f_1=0$. In $\check{Z}$, their classes are respectively $C_3=3\sigma+\pi^*\eta$ and $C_Z\sim C_X\sim C_Y=\sigma$.

MATTER CONTENTS

Accordingly the adjoint **248** of E_8 branches, as summarized in Table I. We identify the fields by charge assignments

$$\begin{aligned} Y&:(\tfrac16,\tfrac16,\tfrac16,\tfrac16,\tfrac16,-\tfrac56),\\ X&:(1,1,1,1,-4,0),\\ Z&:(1,1,1,-3,0,0), \end{aligned} \tag{4}$$

in the basis $\{t_1,t_2,t_3,t_4,t_5,t_6\}$, the weight vectors $\mathbf{5}_1+\mathbf{1}_{-5}$ of $S[U(5)\times U(1)_Y]$. They are localized along curves, the projections of $C_a\cap\tau C_b$ or $C_a\cap C_b, a,b\in\{3,Z,X,Y\}$ on B_2, where τ is involution flipping the orientation of the cover.

The chiral spectrum in four dimension is completed by specifying G-flux. The field strengths along the Cartan direction come from the dimensional reduction of four-form field strength G of the dual M-theory and this induces vector bundle on the spectral cover [5]. Although the minimal $SU(4)$ G-flux preserves unification relation of its commutant $SO(10)$ in E_8, the number of Higgs pairs turns out to be completely fixed to be twice the matter multiplicity [10]. Here we have one more parameter ζ, the trace part of $U(3)_\perp\subset SU(4)$ vector bundle [14], to relax the condition. So we turn on a universal flux

$$\Gamma_3=\lambda(3\sigma-\pi_3^*(\eta-3c_1))+\tfrac13\pi_3^*\zeta,\quad \Gamma_Z=-\pi_Z^*\zeta, \tag{5}$$

where σ is the class for B_2 inside $\check{Z}$ and π_3,π_Z are projections from $U(3)_\perp$ and $U(1)_Z$ covers to B_2, respectively. We do not turn on flux on X and Y cover thus the components t_5 and t_6 can be neglected in calculating the spectrum.

The number n_R of chiral R zero modes minus antichiral $\overline{R}$ ones of the Dirac operator in $\check{Z}$ is a topological number and counted by index theorem. It is simply given by the intersection between matter curve class and Poincaré dual of the G-flux, projected on B_2 [7, 15]

$$n_R=\mathcal{P}_R\cap\Gamma|_{B_2}. \tag{6}$$

Because of identical geometry of spectral cover as in Ref. [16, 17], and we refer to it for the calculation of matter curves

$$\begin{aligned} n_q &= n_{u^c} = n_{e^c} = n_{d^c} = n_l = n_{\nu^c} \\ &= (3\sigma+\eta)\cap\sigma\cap(\lambda(3\sigma_\infty-\eta)+\tfrac{1}{3}\zeta)|_{B_2} \qquad (7)\\ &= (-\lambda\eta+\tfrac{1}{3}\zeta)\cdot(\eta-3c_1), \end{aligned}$$

$$\begin{aligned} n_{D_1} &= n_{h_u} \\ &= -(2\sigma+\eta)\cap(\eta+3\sigma)\cap(\lambda(3\sigma_\infty-\eta)+\tfrac{1}{3}\zeta)|_{B_2} \qquad (8)\\ &= (-\lambda\eta-\tfrac{2}{3}\zeta)\cdot(\eta-3c_1) \end{aligned}$$

$$\begin{aligned} n_{D_2^c} &= n_{h_d} \\ &= (3\sigma+\eta)\cap\sigma\cap(\lambda(3\sigma_\infty-\eta)+\tfrac{1}{3}\zeta-\zeta)|_{B_2}, \qquad (9)\\ &= (-\lambda\eta-\tfrac{2}{3}\zeta)\cdot(\eta-3c_1). \end{aligned}$$

$$n_X = n_Y = n_{T^c} = n_\Sigma = 0, \qquad (10)$$

$$n_Q = n_{U^c} = n_{E^c} = n_{D^c} = n_L = n_{N^c} = c_1\cdot\zeta, \qquad (11)$$

$$\begin{aligned} n_S &= (3\sigma+\eta)\cap\sigma\cap(\lambda(3\sigma_\infty-\eta)+\tfrac{1}{3}\zeta+\zeta)|_{B_2} \\ &= (-\lambda\eta+\tfrac{4}{3}\zeta)\cdot(\eta-3c_1). \end{aligned} \qquad (12)$$

Here we omitted pullback and the product is for the divisors of B_2. We defined $\sigma_\infty = \sigma + \pi^* c_1$. The identities of matter are those in **27** of E_6, and their multiplicities manifest the $SO(10)$ unification relation: it is a nontrivial check that h_u and h_d gives the same number in (8) and (9).

The numbers of matter generations and Higgs pairs can be *individually* controlled, depending on the topological data on B_2. We require three generations of matter and one pair of Higgs doublets

$$\lambda\eta\cdot(\eta-3c_1) = -\tfrac{7}{3}, \quad \eta\cdot\zeta = 2, \quad c_1\cdot\zeta = 0. \qquad (13)$$

However this relation restricts the number of SM neutral field S be five. They are subject to quantization condition $3(\frac{1}{2}+\lambda)\in\mathbb{Z}, (\frac{1}{2}-\lambda)\eta + (3\lambda-\frac{1}{2})c_1 + \frac{1}{3}\zeta \in H_2(S,\mathbb{Z})$. We find a solution $\lambda = \frac{1}{6}$ and the base as del Pezzo two surface with $\eta = 2H, \zeta = H - 3E_1$ do the job, where H is hyperplane divisor and E_1 is one of the exceptional divisor. In addition, because a_1 in (1) transforms as a section of $-c_1$, we have a scalar field O charged under $SU(3)$ and another under $SU(2)$, belonging to $H^{2,0}(B_2) + H^{0,1}(B_2)$ [6]. The other E_8 serves as hidden sector and is completely decoupled in smooth compactification and it can serve as supersymmetry breaking sector.

HIGGS SECTOR AND NUCLEON DECAY

The requirement of *one pair* of Higgs doublets fixed the factorization of spectral cover (3). It has the following phenomenological implications.

Firstly, it also distinguishes between *up and down* type Higgses. This is due to the structure of the $U(3)_\perp$ monodromy S_3 [18], the permutation of the elements $\{t_1, t_2, t_3\}$. It is the natural Weyl group, without a special monodromy further selected by hand. In terms of the S_3 representations, the fields having the same quantum number of lepton doublet under the SM group are

$$\begin{aligned} l &: \{t_1+t_5+t_6, t_2+t_5+t_6, t_3+t_5+t_6\}, \\ h_u^c &: \{t_1+t_2+t_6, t_2+t_3+t_6, t_3+t_1+t_6\}, \\ h_d &: \{t_1+t_4+t_6, t_2+t_4+t_6, t_3+t_4+t_6\}, \\ L &: \{t_1+t_5+t_6, t_2+t_5+t_6, t_3+t_5+t_6\}. \end{aligned} \qquad (14)$$

Effectively, the Higgs doublet is distinguished from the lepton doublet by an opposite matter parity or the $U(1)_X$. It also forbids bare (super)renormalizable lepton and/or baryon number violating operators $lh_u, lle^c, lqd^c, u^cd^cd^c$. Further factorization ruins this one Higgs pair structure but we obtain three pairs of Higgses, so our factorization seems the unique for the $U(n)$ type spectral cover with universal flux.

Well-known is that the matter parity and $U(1)_X$ alone cannot forbid dimension five proton decay operators such as $qqql$ and $u^cu^cd^ce^c$. However, the above structure group *forbids* these operators. For instance, the former is not allowed because of nonvanishing sum of the weights $(t_i)+(t_j)+(t_k)+(t_i+t_5+t_6)$ and $(t_i+t_6)+(t_j+t_6)+(t_k+t_5)+(t_i-t_6)$, required by $U(3)_\perp$ invariance, since one of S_3 index should appear twice [19]. Once forbidden at the tree-level, it is also known that the induced operators are highly suppressed, probably explained by worldsheet instanton contribution [20].

Another prediction is the presence of an SM singlet field S. Surveying the quantum number, it belongs to **27** representation of E_6, therefore, its interaction is restricted and we can calculate the corresponding terms. Because Higgs doublet and triplets are not simply vectorlike and up and down Higgses live on different matter curves, bare masses are forbidden by $U(3)_\perp$ invariance. Instead we have Next-to-Minimal Supersymmetric SM (NMSSM) [21, 22]. We can check that the only renormalizable superpotential for the surviving fields are (see also below)

$$\begin{aligned} &qh_du^c + qh_ud^c + lh_de^c + lh_u\nu^c + Sh_uh_d + SD_1D_2^c \\ &\quad + qqD_1 + u^ce^cD_1 + qlD_2^c + \nu^cd^cD_1 + u^cd^cD_2^c \end{aligned} \qquad (15)$$

omitting the flavor dependent coefficients. We expect the terms involving D_1 and D_c^2 are all decoupled, yielding the μ-like term Sh_uh_d. Bare quadratic or cubic terms in S are not allowed. Induced higher order terms include $M_s^{-2}SS(QD_2^cL + D^cND_1 + U^cE^cD_1) + M_s^{-4}SSS(QU^cE^cL + QD^cN^cL)$ but they are to be suppressed by a string scale M_s. A Majorana mass for the ν^c does not appear up to dimension five. There is an interesting room for this from Euclidean D3-brane or gauge instanton in F-theory [23], which might as well generate similar potential for S. Remarkably, if one combination $X-Z$ of $U(1)_X$ and $U(1)_Z$ is broken down, the orthog-

onal combination $X+Z$ inducing a Majorana neutrino mass also generates S^3 term $e^{-S_{\rm inst}}(\nu^c\nu^c+SSS)$.

Since the Higgs fields also obey $SO(10)$ unification relation, we have as many colored Higgs pairs D_1, D_2^c as doublets. This doublet-triplet splitting problem should be solved by an effect evading the unification structure, close but below the compactification scale. It is an interesting possibility to consider a *vectorlike* extra generation of matter fields, without changing the Dirac indices. Using aforementioned $SU(3)$ scalar O, there can be terms $\langle O\rangle D_1D_1^c+\langle O\rangle D_2D_2^c+M_O{\rm tr}\,O^2+{\rm tr}\,O^3$ giving Dirac masses separately to D_1, D_1^c and D_2, D_2^c pairs. Conventional gauge coupling unification requires heavy triplets, so do a large VEV $\langle O\rangle$ and a large M_O. On the other hand, we expect a coupling $(\langle S\rangle+\mu_D)D_1D_2$ is generated, with a possible SUSY breaking effect μ_D. The most strongly constrained nucleon decay operator is $qqql$, whose coefficient has upper bound $10^{-5}M_P^{-1}$ [25]. At low energy scale, integrating out heavy fields, qqD_1 and qlD_2^c may induce an operator $(\langle S\rangle+\mu_D)/M_D^{-2}qqql$ up to geometric suppression factor. If the numerator is naturally small or tuned, we can suppress this operator. A possible mixing from bare mass term d^cd does not change this eigenvalue. The same argument goes to other induced operators for nucleon decay.

GAUGE COUPLING UNIFICATION

In type IIB language, Kaluza–Klein expansion of four-form field along the harmonic two-forms w_i on B_2, $C_4=C_2\wedge w_2$, we have a term ${\rm tr}\,Q^2\int_{M^4}F_Q\wedge C_2\int_{B_2}w_2\wedge\iota F_Q$ from Chern–Simons interactions and here F_Q is the field strength for $U(1)_Q$ flux and ι is immersion to B. We turn on is for $U(1)_Z$ thus the corresponding gauge boson acquire mass by Stückelberg mechanism and the symmetry is broken. We do not turn on flux along hypercharge or X direction, so they are not broken and their gauge couplings receive no threshold correction from the flux [26, 27]. The existence of $U(1)$'s is clearly seen in the heterotic dual side. Four gauge couplings unify with the correct normalization in $SO(10)$

$$g_3=g_2=\sqrt{5/3}g_Y=\sqrt{40}g_X,\ \sin^2\theta_W=3/8,$$

at M_s. This is the consequence of the inheritance principle, for all the gauge group is embedded in E_8 and hence $SO(10)$. The $U(1)_X$ can survive at relatively low energy scale and be spontaneously broken down at relative low energy. Threshold corrections for the splitted Higgs triplets would modify the scale.

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* kschoi@kias.re.kr

[1] H. Georgi, S. L. Glashow, Phys. Rev. Lett. **32**, 438-441 (1974); J. C. Pati, A. Salam, Phys. Rev. **D10**, 275-289 (1974).

[2] P. Langacker, J. Wang, Phys. Rev. **D58**, 115010 (1998); S. F. King, et. al., Phys. Rev. D **73**, 035009 (2006).

[3] See also J. J. Heckman, A. Tavanfar and C. Vafa, arXiv:0906.0581 [hep-th].

[4] H. P. Nilles, [hep-th/0410160]; R. Donagi, M. Wijnholt, [arXiv:0808.2223 [hep-th]]; J. J. Heckman and C. Vafa, JHEP **0909** (2009) 079.

[5] R. Friedman, J. Morgan and E. Witten, Commun. Math. Phys. **187**, 679 (1997).

[6] R. Donagi and M. Wijnholt, arXiv:0904.1218 [hep-th].

[7] H. Hayashi, R. Tatar, Y. Toda, T. Watari and M. Yamazaki, Nucl. Phys. B **806** (2009) 224.

[8] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901** (2009) 058; JHEP **0901** (2009) 059.

[9] T. W. Grimm, Nucl. Phys. **B845**, 48-92 (2011).

[10] K.-S. Choi and T. Kobayashi, arXiv:1003.2126 [hep-th].

[11] K.-S. Choi, Nucl. Phys. **B842**, 1-32 (2011).

[12] M. Bershadsky, K. A. Intriligator, S. Kachru, D. R. Morrison, V. Sadov, C. Vafa, Nucl. Phys. **B481**, 215-252 (1996); S. Katz, D. R. Morrison, S. Schafer-Nameki, J. Sully, JHEP **1108**, 094 (2011).

[13] J. Marsano, N. Saulina and S. Schafer-Nameki, JHEP **0908**, 046 (2009).

[14] R. Blumenhagen, G. Honecker and T. Weigand, JHEP **0506** (2005) 020.

[15] R. Donagi, Y. H. He, B. A. Ovrut and R. Reinbacher, JHEP **0412** (2004) 054; R. Blumenhagen, S. Moster, R. Reinbacher and T. Weigand, JHEP **0705**, 041 (2007).

[16] K.-S. Choi, B. Kyae, [arXiv:1102.0591 [hep-th]].

[17] C. -M. Chen, Y. -C. Chung, JHEP **1103**, 049 (2011); JHEP **1103** (2011) 129.

[18] R. Tatar, Y. Tsuchiya and T. Watari, Nucl. Phys. B **823** (2009) 1.

[19] See also C. Ludeling, H. P. Nilles, C. C. Stephan, Phys. Rev. **D83**, 086008 (2011); H. M. Lee, et. al., Nucl. Phys. **B850**, 1-30 (2011); K. -S. Choi, J. E. Kim, Phys. Rev. **D83**, 065016 (2011).

[20] For example, P. Cámara's talk at String Pheno 2011. See also K.-S. Choi and T. Kobayashi, Nucl. Phys. B **797** (2008) 295; H. Abe, et. al., JHEP **0906** (2009) 080.

[21] U. Ellwanger, C. Hugonie, A. M. Teixeira, Phys. Rept. **496**, 1-77 (2010) and references therein; See also O. Lebedev, S. Ramos-Sanchez, Phys. Lett. **B684**, 48-51 (2010).

[22] M. Cvetic, J. Halverson, P. Langacker, JHEP **1009**, 076 (2010).

[23] R. Blumenhagen, A. Collinucci, B. Jurke, JHEP **1008**, 079 (2010); T. W. Grimm, M. Kerstan, E. Palti, T. Weigand, [arXiv:1105.3193 [hep-th]]; M. Cvetic, J. Halverson, I. Garcia-Etxebarria, [arXiv:1107.2388 [hep-th]].

[24] G. F. Giudice, A. Masiero, Phys. Lett. **B206**, 480-484 (1988). J. E. Kim, H. P. Nilles, Phys. Lett. **B138**, 150 (1984).

[25] S. Raby, [hep-ph/0211024].

[26] R. Blumenhagen, Phys. Rev. Lett. **102** (2009) 071601.

[27] T. W. Grimm and T. Weigand, arXiv:1006.0226 [hep-th].

[28] K.-S. Choi, JHEP **1002**, 004 (2010).